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YIELD STRESS UNDER REPEATED LOADING AFTER APPLYING DIFFERENT DEFORMATION PATH FOR LARGE TENSION AND SHEAR

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This paper describes the yield phenomenon when a repeated loading is applied to the structure after giving a large pre-strain on it. In the series of our previous research, focusing on the fundamental deformations such as tension or shear, changes of yield stress as the number of repetition increases have been investigated experimentally by using test specimens of annealed pure copper. In the determination of yield stress value, the method by using slope of stress-strain curve at yield has been used instead of proof stress. As a consequence, it has been found that if the type of pre-deformation and the type of deformation in repeated load are the same, yield stress at the pre-deformation side has a declining tendency as compared with opposite side. Therefore, it is predicted that the yield stress under repeated loading after applying a large deformation is closely related to the loading history in large deformation previously applied. Thus, in this study, in order to clarify the influence of pre-deformation on the yield behavior under repeated loading, the experiments are performed by changing the order of tension and shear in pre-deformation. Consequentially, it is clarified that the declining tendency of yield stress under repeated loading is closely related to the yield surface anisotropy, which is formed during the second half of the pre-deformation.

Keywords: Finite strain, Yield phenomenon, Natural strain, Anisotropy, Deformation history, Ductile metal.

1 INTRODUCTION

With regard to the yield phenomena under repeated loads, many research studies have been widely conducted since long before, however, most of them deal with cyclic loads measured from initial small deformation, and the yield phenomena after large plastic deformation are not necessarily elucidated. When the repeated loads act on the structure after applying large deformation, just as earthquake, the plastic fracture occurs in the structure despite low number of repetitions. Therefore, it is an important subject to elucidate the low cycle fatigue and the elastoplastic behavior under repeated loading after a large deformation acts on a material.

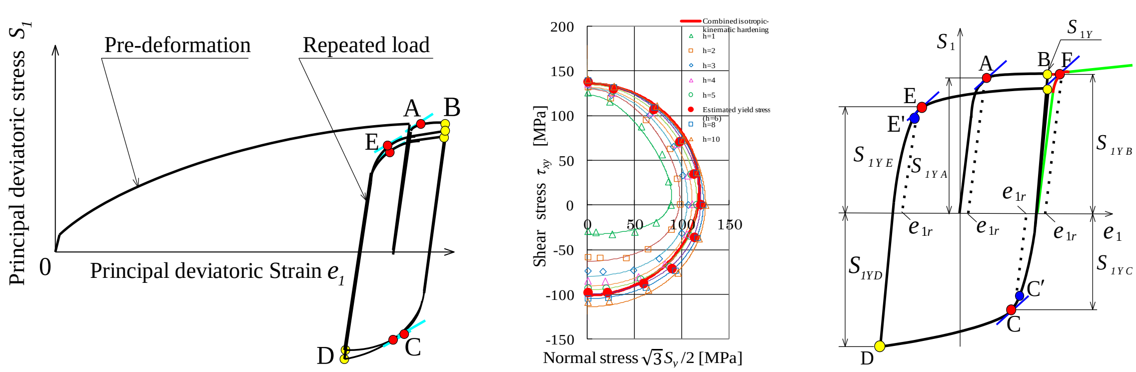
Thus, in the present study, the plastic behavior under repeated loading after large deformation is elucidated by using the Natural Strain theory. Here, the Natural Strain in this research, the shear strain is defined by a pure rotating angle of identical line element. Thus, the rigid body rotation is completely removed from shear strain component. Moreover, since the associative and commutative laws for strain can be satisfied, the elastic component and the plastic component of the strain rate are decomposed clearly, and further, deviatoric and volumetric parts are also decomposed in the same way as infinitesimal deformation theory. Thus, since this strain theory is rational and useful for expressing a large plastic deformation, the elastoplastic behavers are elucidated on the basis of this theory in the present research.

In our research, the yield stress under repeated loading has been investigated focusing on fundamental deformations such as tension or shear. In those studies, the yield stress has been investigated under the conditions that the types of deformation applied in pre-deformation and in repeated loads are the same and these are the uniaxial state (Kato 2017). And in the next stage, the yield stresses have been investigated also to the experimental conditions where the types of deformation applied in the pre-deformation and in the repeated loads are different. From those experimental results, it has been revealed that since the effect of the anisotropy formed in the pre-deformation gradually decreases during the process of repeated loads, the yield stresses on the side where the pre-deformation acts decrease and gradually approach to constant value, and moreover, these tendencies are closely related by the difference of deformation types in pre-deformation and in repeated loads (Kato and Uchida 2018).

In the present research, focusing on the pre-deformation history, the experiments are performed by changing the order of tension and shear in the pre-deformation. Then, the effects of loading path in the pre-deformation on the decreasing tendencies of yield stress during repeated loads are investigated.

2 LOADING PATH IN PRE-DEFORMATION AND YIELD STRESS IN CYCLIC LOAD

In this study, the changes of yield stress in repeated loading tests have been investigated by using test specimens that a large deformation has been applied in advance as shown in Figure 1 (a), rather than repeated loads are applied from small deformation. During the process of giving a pre-deformation, the anisotropy concerning the yield surface is formed as shown in Figure 1 (b). The curves in this figure mean the locus where the hardening modulus becomes the same value. Here, the distance between these curves is narrower on the side where the pre-deformation acts, on the other hand, it becomes wider at the opposite side. Hence, the back stress is generated on the side where the pre-deformation acts and the anisotropy is generated during a large deformation. However, the influence of anisotropy caused by large deformation is gradually reduced when the repeated loads are applied. As a consequence, it is predicted that the yield stress value in each cycle changes as the number of repetitions increases as shown in Figure 1 (a). In this research, in order to investigate the influence of pre-deformation history on the yield stress value in repeated loading, the experiments are conducted under different experimental conditions described later.

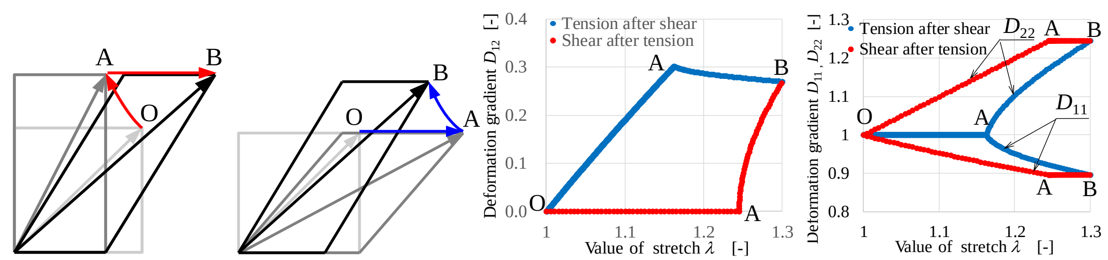


(a) Repeated loads after large deformation (b) Anisotropy in yield surface (c) Yield stress in first cycle

Figure 1. Relation between the large pre-deformation and repeated loads.

2.1 Effect of Deformation History on Yielding Phenomena During Repeated Loads

It has been revealed form our previous study that the anisotropy is formed during the second half of the pre-deformation and is closely related to the declining tendency of yield stress in repeated loading tests (Kato and Uchida 2018). Hence, in this research, the pre-deformations of different deformation path are examined for elucidating the influence of the pre-deformation history on the declining tendency of yield stress in repeated loads in more detail. Here, Figure 2 (a), (b) show the schematic diagrams of deformation path for combined loading state that are explored in this research, and Figure 2 (c), (d) show the relations between the corresponding deformation gradient and the principal stretch λ. In Figure 2 (c), (d), the red curves represent the deformation paths for shear after tension and the blue curves are the deformation paths for tension after shear, respectively. However, these pre-deformations are given so that the final state (namely, point B in these figures) becomes the same value. It is predicted that the anisotropy concerning the yield surface progress mainly in the latter half of these paths (namely, deformation path between AB). Here, Figure 3 shows relations between the directions of the back stress corresponding to each path and the directions of repeated loading. In the case of (α- I) and (α- II), since the latter half of deformation path is shear deformation, the kinematic hardening occurs largely on the shear side, i.e., the direction of green arrows. In contrast to this, in the case of (β- I) and (β- II), since the latter half of deformation path is tensile deformation, the kinematic hardening occurs largely on the tension side. On the other hand, yellow arrows in these figures represent the directions of cyclic loads, (α- I) and (β- I) indicate the directions of cyclic loads for forward and reverse shear, and (α- II) and (β- II) are the directions for tension and compression, respectively. Therefore, in the present study, the yield stresses in repeated loading tests are examined under these different combinations.



(a) Shear after tension (b) Tension after shear (c) Deformation gradient D_{12} (d) Deformation gradient D_{11},D_{22}

Figure 2. Deformation path in pre-deformation.

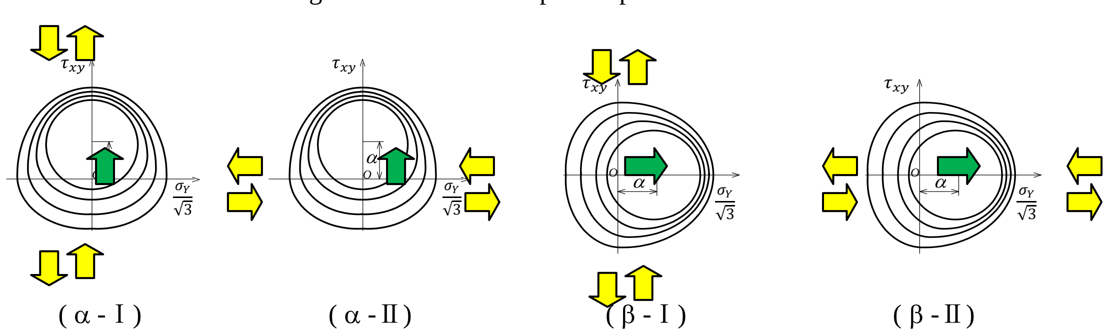


Figure 3. Relation between anisotropy in yield surface and repeated loading.

2.2 Yield Stress Under Repeated Loads After Pre-Deformation of Different History

Figure 1 (c) shows the deviatoric stress-strain curve of the repeated loading tests. Here, the green curve in Figure 1 (c) shows stress behavior at re-yielding. Since the stress at point B is determined in advance, the value of yield stress on the green curve has already been known. Thus, in this figure, a point F is considered to be the yield stress. In the vicinity of the point F, the deviatoric stress-strain curve becomes a gently curve, and in our previous study (Kato 2017, Kato 2018), it is expressed as shown in Eq. (1).

$$S_1 = a\left\{1 - \exp(b e_1)\right\} + c e_1 + d \tag{1}$$

where, *a*, *b*, *c* and *d* which shown in Eq. (1) are derived by using the *Levenberg-Marquardt Method* (Levenberg 1944). Furthermore, the slope of tangent is also formulated as shown in Eq. (2).

$$\frac{d S_1}{d e_1} = -a\, b\, \exp\left(b e_1\right) + c \tag{2}$$

Thus, the slope at re-yielding can be obtained in advance by Eq. (2), and in this figure, the straight blue line through a point F represents the slope at re-yielding. In this study, it will be assumed that the stress with the same slope as point F is regarded as the yield stress. Therefore, the estimated yield stresses are represented by $S_{1y\,\mathrm{C}}$ and $S_{1y\,\mathrm{E}}$ (see points C and E). On the other hand, the yield stresses just before unloading, which are obtained after applying constant strain amplitude in the forward and reverse directions, are $S_{1y\,\mathrm{B}}$ and $S_{1y\,\mathrm{D}}$ (see points B and D).

3 EXPERIMENTAL METHOD AND CONDITIONS

The multiaxial testing machine which can apply axial and torsional displacements simultaneously was adopted. Among ductile materials, annealed test specimens made of tough pitch copper (purity 99.9%) with a cylindrically shaped cross-section (namely, outer diameter is 22 [mm], inner diameter is 16 [mm], and gauge length is 30 [mm]) are adopted in the experiments.

This experiment consists of two kinds of experiments. One is the large pre-deformation of different deformation path. The other one is repeated loading tests.

(i) Test conditions for pre-deformation of different deformation path
The experiments are performed on the following two deformation paths:
(a) The tension, i.e., the principal stretch of m = 1. 245 [-], is applied first. Then, the torsion, i.e., the shear deformation of D_{12} = 0.27 [-] (torsional angle φ = 38.2 [deg.]), is applied under the principal stretch at the final deformation states is λ =1.3 [-].
(b) The torsion, i.e., D_{12} = 0.3 (torsional angle φ = 54.3 [deg.]), is applied first, and then, the tension, i.e., m = 1.245 [-], is applied under the condition that the principal stretch at the final deformation states is λ =1.3 [-]. However, the reverse torsion occurs during tension.

(ii) Conditions for repeated loading tests

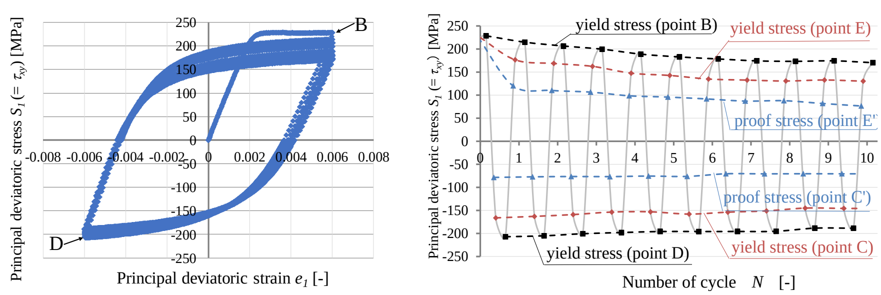
Attaching tri-axial strain gauge to test specimens, the repeated loading tests for tension and compression or forward and revere shear are carried out under the same strain amplitude $\Delta e_1 = \pm$ 0.006[-]. However, the numbers of cycles are the same condition, i.e., N =10.

4 EXPERIMENTAL RESULTS

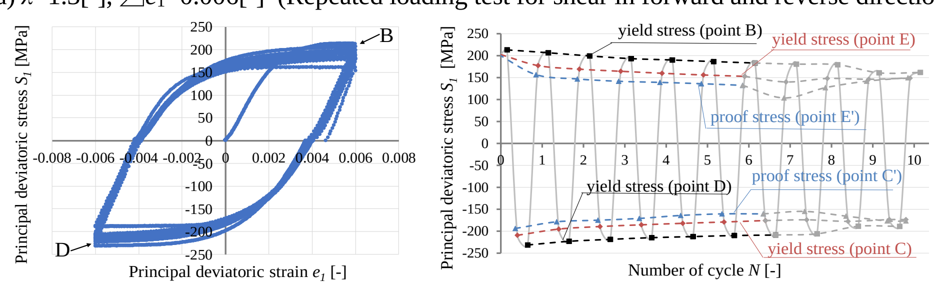
Figure 4 show the principal deviatoric stress - strain diagram and the yield stress which varies with the number of repetitions for the case of pre-deformation concerning shear after tension indicated in Figure 2 (a). Here, Figure 4 (a) shows results derived by repeated loading tests for shear in the forward and reverse direction and Figure 4 (b) shows results derived by repeated loading tests for uniaxial tension - compression. In these figures, the yield stresses for point B and point D in Figure 1(c), i.e., $S_{1\text{YB}}$ and $S_{1\text{YD}}$ are indicated by black plots, and the yield stresses for points C and E, i.e., $S_{1\text{YC}}$ and $S_{1\text{YE}}$ are indicated by red plots. On the other hand, the results derived by proof stress for points C' and E' in Figure 1(c) are represented by blue plots. Thus, it can be seen from these figures that the yield stresses obtained in this study are higher value as compared to the proof stress.

Here, when focusing on the declining tendency of yield stress for point B and point D, in the case of the Figure 4 (a) that the types of deformation in latter half of pre-deformation and in cyclic loads are the same, the reduction of the yield stress for point D become smaller than the declining tendency of yield stress concerning point B. On the other hand, in the case of Figure 4 (b) that the types of deformation in latter half of pre-deformation and in cyclic loads are different, the reduction of the yield stress for point D and point B is almost the same. Moreover, in this case, it can be confirmed that buckling has already begun to occur before 7th cycle, see the gray lines in this figure.

Next, Figure 5 (a), (b) represents the experimental results for the repeated loading tests after giving pre-deformation concerning tension after shear indicated in Figure 2 (b). Similarly, Figure 5 (a) shows results derived by repeated loading test for shear in forward and reverse direction and Figure 5 (b) shows results by repeated loading test for tension - compression. In the case of the Figure 5 (a) that the deformation in latter half of pre-deformation and the deformation in repeated loads are different type, the reduction of yield stress for point D and point B is almost the same. On the other hand, in case of the Figure 5 (b) that these types are the same, the reduction of the yield stress for point D becomes smaller than the declining tendencies of yield stress for point B.

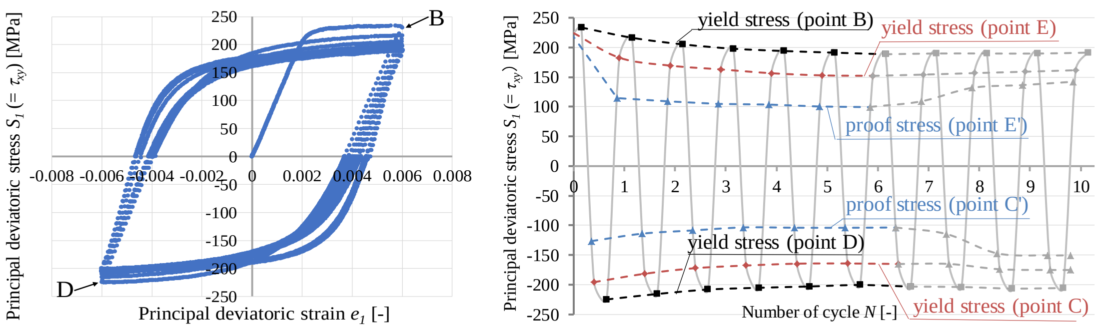


(a) λ=1.3[-], $\triangle e_1$=0.006[-] (Repeated loading test for shear in forward and reverse direction)

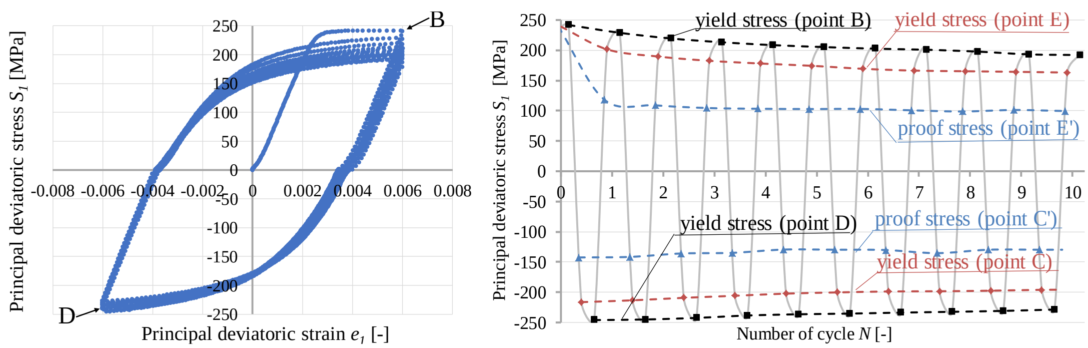


(b) λ=1.3[-], $\triangle e_1$=0.006[-] (Repeated loading test for uniaxial tension - compression)

Figure 4. Yield stress obtained by repeated loading tests after pre-deformation concerning shear after tension.



(a) λ=1.3[-], $\angle e_1$=0.006[-] (Repeated loading test for shear in forward and reverse direction)



(b) λ=1.3[-], $\angle e_1$=0.006[-] (Repeated loading test for uniaxial tension - compression)

Figure 5. Yield stress obtained by repeated loading tests after pre-deformation concerning tension after shear.

5 CONCLUSIONS

(1) In the early stages of cycles, there is a declining tendency in yield stresses. However, as the number of repetitions increases, the declining tendency hardly appears, and the yield stresses becomes constant value.

(2) If the deformation giving in latter half of pre-deformation and the deformation in repeated loading tests are the same type, the reduction of the yield stress with respect to point D in cycle becomes smaller than the declining tendency for point B. On the other hand, if these types are different, the reduction of the yield stress for point D is almost the same as point B.

(3) Moreover, if the types of deformation in latter half of pre-deformation and in cyclic loads are different, buckling occurs after 7 cycles.

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